Interactions between a rotating polarized sphere and a stationary one in an electric field

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(Received 28 April 2005; published 20 October 2005)

Precise measurement of the attracting force between two polarized spheres inside an electric field indicates that the rotation of one sphere along the axis perpendicular to the electric field reduces the attracting force between them. The important difference between the experimental results and the existing theory indicated that this reduction is due to several factors. In addition to the reduction of polarization due to the free surface charges, the rotation may also weaken the local field near the rotating sphere, making the main contribution to the reduction of the attracting force. Moreover, the experiment also suggests that the polarization due to the molecular polarizability cannot be ignored.

DOI: 10.1103/PhysRevE.72.041508 PACS number(s): 83.80.Gv, 77.22.Ej, 41.20.−q

I. INTRODUCTION

Interactions between rotating dipoles are very important in the study of electrorheological (ER) and magnetorheological (MR) fluids since the particles in these fluids may rotate under a shearing flow [1–10]. This topic has recently attracted much attention. In a series of papers [11–13], Wan, Yu, and Gu (WYG) proposed a theory claiming that the rotation leads to a redistribution of the polarization charges on the rotating particle surface and hence reduces its dipole moment. However, up to date, there has been no direct comparison between these theoretical calculations and experimental results. In fact, it is very difficult to compare any experimental results in ER and MR fluids with their theoretical predictions since the rotation of particles in ER or MR fluids induces significant hydrodynamic force, which may affect the effective viscosity and weaken the binding between the dipoles. Unfortunately, it is virtually impossible in ER and MR experiments to separate the electric and magnetic effects and the hydrodynamic effect caused by the same rotation.

To clarify the issue, here we report our experiment, which directly measured the attracting force between two polarized spheres in argon gas closely aligned in the applied electric field. One of the spheres was rotating along an axis perpendicular to the electric field and the other was stationary. This experiment should be directly comparable with the WYG theory as the hydrodynamic force is negligible.

According to the WYG theory, as a dipole rotates at angular speed ω in the direction perpendicular to the applied field, the component of its dipole moment along the field direction is reduced by a factor $1/[1+(\omega\tau)^2]$, where τ is the relaxation time given by

$$\tau = (\varepsilon_p + 2\varepsilon_f)/(\sigma_p + 2\sigma_f). \tag{1.1}$$

We use ε_p and ε_f (σ_p and σ_f) to denote the dielectric constant (conductivity) of the rotating sphere and host medium, respectively. Therefore, the attracting force between our two spheres should also be reduced by a factor $1/[1+(\omega\tau)^2]$. The WYG theory also claims that this reduction rate is independent of the gap between the two dipoles.

While our experiment found that the interaction was reduced when one sphere rotated, there were significant differences between the WYG theory and our experimental results. In the case of a rotating copper sphere, the relaxation time estimated from Eq. (1.1) is extremely small and $\omega\tau \ll 1$ in our experiment. The WYG theory would suggest that the rotation in our experiment should have little effect on the attracting force. In fact, our experiment finds that the reduction is significant. In the case of a rotating polyamide sphere, we estimate $\omega\tau \gg 1$ from Eq. (1.1). The attracting force should tend to zero as fast as $1/[1+(\omega\tau)^2] \approx 1/(\omega\tau)^2$. Instead, our experiment seems to suggest that the attracting force is reducing at a much slower pace. From the above results, we have to conclude that the WYG theory is insufficient in describing the interactions between rotating spheres and must be modified.

The current paper is organized as follows. We will report the experimental setup in Sec. II and present detailed experimental results in Sec. III. Some theoretical explanations and suggestions for modification of the existing theory are in Sec. IV.

II. EXPERIMENTAL SETUP

The device for the experiment is sketched in Fig. 1. Two spheres of diameter 1.906 cm were arranged vertically inside a capacitor made of two horizontal electrodes separated by 20 cm. A high voltage up to 50 kV was applied to the electrodes, producing a uniform electric field inside. The bottom sphere was rested on a microbalance. If there is no electric

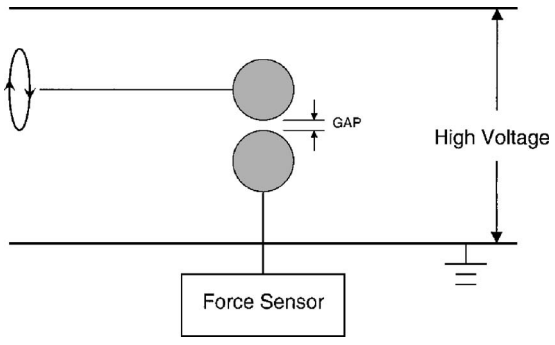

FIG. 1. The experimental setup.

field, the reading on the microbalance was just the bottom sphere's weight. In the electric field, the two spheres, made of either cooper or polyamide, were polarized. The induced attracting force between them reduced the reading on the microbalance. Hence we determined this attracting force directly. The top sphere, which could move up or down to adjust the gap between the spheres, was connected to a motor and able to rotate around the horizontal axis up to 2750 rotations per minute (rpm).

Since the measurement was very delicate, we took many precautions. During our preliminary test, we found that because of the high local electric field between the two spheres, some air molecules could be easily ionized when the top sphere rotated at a high speed. In a dc field, these ionized charges would attach to the two spheres, producing additional force and making the precise measurement impossible. In addition, if there were ionized charges attached to the sphere surface, the charges did not diminish and the force between them remained after the electric field was turned off. As a result, the readings on the microbalance after the electric field was turned off could not match the bottom sphere's weight. Meanwhile, we also found that it was easy to have dielectric breakdowns inside a vacuum. To solve all these problems, we placed the whole device inside a glove box, which was vacuumed first and filled with dry argon gas afterwards. The inertial gas reduced the chance of dielectric breakdown and the electric field between the spheres was not strong enough to ionize argon molecules. We further switched to a low-frequency ac electric field in our measurement, instead of a dc electric field. These measures worked very well. After the electric field was turned off, the reading on the microbalance returned to the bottom sphere's weight precisely. This was an indication that no ionized charges were attached to the spheres.

The hydrodynamic force on the spheres due to argon gas was also an issue to be considered. We carefully examined but did not find any change in the reading on the microbalance when the top sphere rapidly rotated with no electric field applied. It appeared that because the argon gas had very low viscosity and the flow inside the gap caused by the rotation mainly was in the horizontal direction, the vertical hydrodynamic force was too weak to be detected. The measurement of attracting force in the vertical direction was not affected by the argon gas flow.

III. EXPERIMENTAL RESULTS

A. Rotating metallic sphere: Stationary metallic sphere

We first carried out experiments with two copper spheres. As shown in Fig. 2(a), the attracting force decreased rapidly as the gap increased. In addition, if the frequency of the applied ac electric field got higher, the attracting force got smaller. As shown in Fig. 2(b), the attracting force was proportional to E_0^2. The finite-element analysis estimated the following attracting force for two metallic spheres in a dc electric field [14]:

$$f = -\varepsilon_f E_0^2 a^6 (6d - 4a)/[d^4(d - 2a)], \tag{3.1}$$

where d is the distance between the two sphere centers, a is the sphere radium, and ε_f is the dielectric constant of media—i.e., the argon gas here. Our experiment has found that in a low-frequency ac field, the attracting force was close to

$$f = -\alpha(\omega_e)\varepsilon_f E_0^2 a^6 (6d - 4a)/[d^4(d - 2a)], \tag{3.2}$$

where the constant $\alpha(\omega_e)$ decreased with the frequency of the applied field ω_e and was smaller than 1.

As shown in Fig. 3, if the top sphere rotated, the attracting force was reduced. The higher the rotation speed, the more the attracting force decreased. However, as the gap increased, the rotational effect weakened. For example, for $E = 100$ V/mm at gap=0.254 mm, the force was 75.6 dyn at no rotation and down to 55.8 dyn at 2750 rpm, a reduction of 26%; however, at gap=0.381 mm, the force was 53.4 dyn at no rotation and down to 55.8 dyn at 2750 rpm, a reduction of 9% only. We also noted that as the rotation speed increased, the reduction got slow. For example, for $E = 100$ V/mm at gap=0.254 mm, from no rotation to 660 rpm, the force was down from 75.6 dyn to 62.9 dyn, a 16.8% reduction; however, from 1750 rpm to 2750 rpm, the force was down from 58.9 dyn to 55.8 dyn, a reduction of 5.2% only. In Fig. 4 we plot the attracting force versus $1/\omega$.

While our experiment verified that the dipole's rotation reduced the dipolar interaction, there were important differences between our experimental results and the WYG theory. For the rotational copper sphere, the relaxation time τ is estimated about 10^{-17} s from Eq. (1.1). Then even at the highest rotation of 2750 rpm in our experiment, we only had $\omega\tau = 2.88 \times 10^{-15}$. If the prediction was correct, there was no way for us to detect any change in the attracting force since the reduction was only of order of 10^{-29}, $1/[1+(\omega\tau)^2]-1 = 8 \times 10^{-30}$. In fact, our experiment found that the reduction was significant.

In addition, the WYG theory predicts that the reduction rate $1/[1+(\omega\tau)^2]$ is independent of the gap between the two spheres. However, our results indicated that the reduction rate depended on the gap between the two spheres. If the gap was smaller, the reduction rate was faster. This gap dependence indicates that the main reduction of the attracting force is due to the change of local field between the two spheres when one of them rotates. The WYG theory did not address this issue.

B. Rotating metallic sphere: Stationary dielectric sphere

After we replaced the lower stationary copper sphere by a dielectric sphere, the attracting force was considerably reduced as shown in Fig. 5. For example, for $E = 100$ V/mm and gap=0.254 mm, the force was 75.6 dyn for two metallic spheres at no rotation and only 4.3 dyn now. The rotation reduced the attracting force as significantly as in the case of two metallic spheres. For example, for a gap=0.254 mm and $E = 100$ V/mm, from 0 rpm to 2750 rpm, the force was down from 4.3 dyn to 3 dyn, a reduction of 30%, compared with 26% for two metallic spheres. Similarly, as the gap increased, the rotational effect weakened. For example, for $E = 100$ V/mm at gap=0.508 mm the force was 3.4 dyn at no rotation and down to 2.9 dyn at 2750 rpm, a reduction of 14.7%, compared with 30% for the case of gap=0.254 mm.

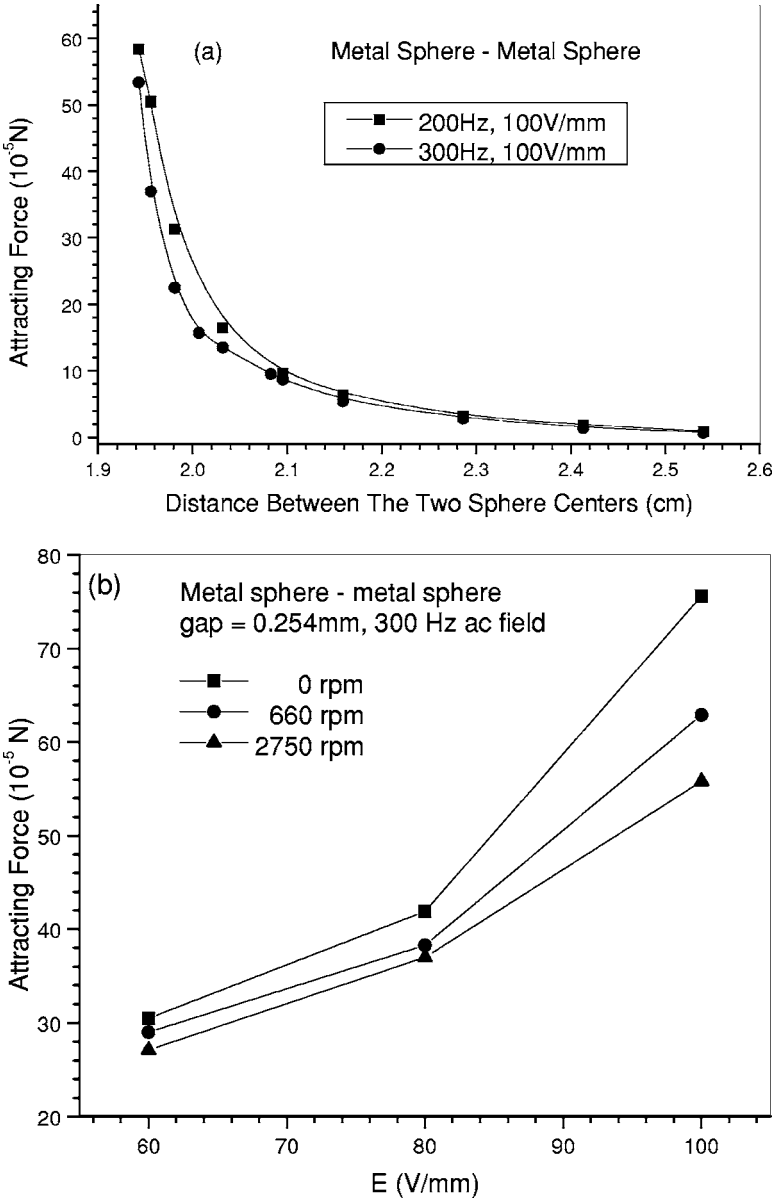

FIG. 2. The attracting force between two metallic spheres. (a) The relationship between the force and the gap. (b) The relationship between the force and electric field.

We also noted that as the rotation speed further increased, the reduction got slower. For example, for $E=100$ V/mm at gap=0.254 mm, from no rotation to 660 rpm, the force was down from 4.3 dyn to 3.8 dyn, a 11.6% reduction; however, from 1750 rpm to 2750 rpm, the force was down from 3.2 dyn to 3.0 dyn, a reduction of 6.3% only. We plotted the attracting force versus $1/\omega$ in Fig. 6.

Since the rotating sphere was made of copper in our experiment, according to the WYG theory, the relaxation time τ was still 10^{-17} s and the reduction of the attracting force remained as $1/[1+(\omega\tau)^2]$, negligible. Similarly, the experiment found again that the reduction rate depended on the gap, a clear indication that this reduction was mainly from the weakening of the local field as a result of the rotation of the metallic sphere.

C. Rotating dielectric sphere: Stationary dielectric sphere

After we replaced both copper spheres by dielectric spheres, the attracting force further reduced. As shown in Fig. 7, for example, for $E=100$ V/mm and gap=0.254 mm, the force was 1.8 dyn now for both dielectric spheres, compared to 75.6 dyn for two metallic spheres and 4.3 dyn for one metallic sphere and one dielectric sphere. The rotation reduced the attracting force significantly. As the gap increased, the rotational effect got weak, but at a speed much slower than that in two previous cases. For example, for $E=100$ V/mm at gap=0.254 mm, the force was 1.8 dyn with no rotation and down to 1.4 dyn at 2750 rpm, a reduction of 22%; however, at gap=0.381 mm for the same electric field, the force was 1.0 dyn at no rotation and down to 0.8 dyn at 2750 rpm, a reduction of 20%, slightly smaller than 22%. To see the behavior in $\omega\rightarrow\infty$, we plotted the attracting force versus $1/\omega$ in Fig. 8.

Since polyamide has resistivity around 10^{12} Ω cm, the relaxation τ was estimated in the order of seconds from Eq. (1.1). At the highest rotation of 2750 rpm, we had $\omega\tau=288\gg1$ in our experiment. The attracting force should tend to zero as fast as $1/[1+(\omega\tau)^2]\approx1/(\omega\tau)^2$. Instead, as shown

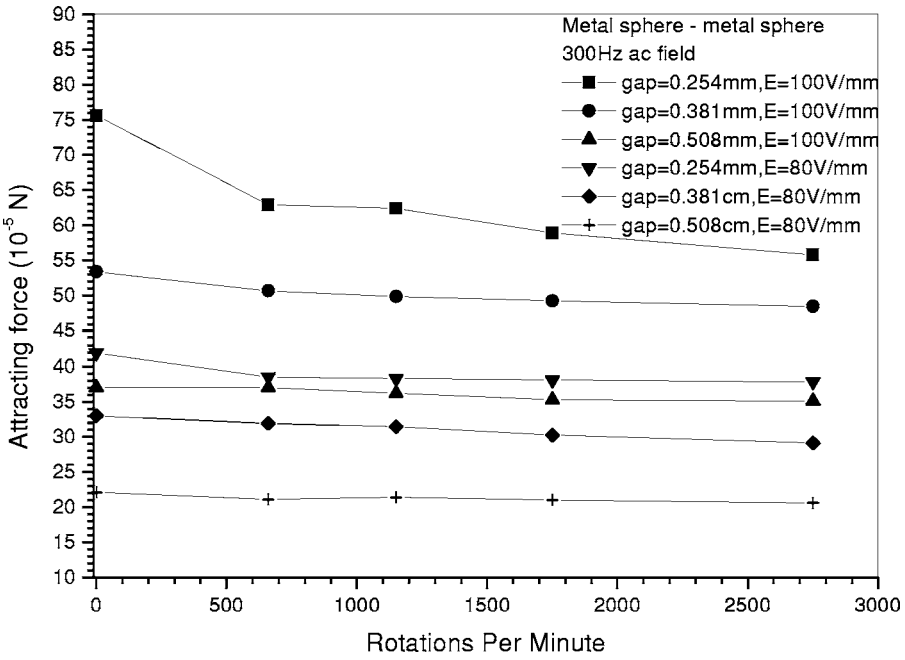

FIG. 3. The attracting force between two metallic spheres versus the rotation speed.

in Fig. 8 the attracting force was reducing at a much slower pace. This indicates that the WYG theory is incomplete in considering the reduction mechanism.

The experiment also found that the reduction rate of the attracting force weakly depended on the gap in this case. This is consistent with the following picture. A rotating dielectric sphere also weakens the local field at its neighborhood, but it does not affect the local field as strongly as a conducting sphere.

IV. DISCUSSION AND CONCLUSIONS

The microbalance used in our experiment had a readability of 0.01 mg, which was about 10^{-7} N. Therefore, the microbalance was not a significant source for error here. The main sources for error were the measurement of the gap and the voltage applied. The mechanical gauge to measure the gap could have a 2%–3% error while the applied voltage could have a 2%–3% error. Therefore, all together, the error bar for our experimental results was about 5%.

The important differences between our experimental results and the WYG theory lead us to believe that the WYG theory is insufficient and must be modified. Here, we outline our suggestions for a theoretical explanation and possible modifications for the WYG theory.

The current WYG theory only considers the redistribution of the free surface charge on the rotational sphere. It is clear now that this is insufficient. As stated before, when the rotating sphere is metallic, $\omega\tau \leqslant 2.88\times10^{-15}$; if the prediction by the WYG theory was correct, there was no way for us to

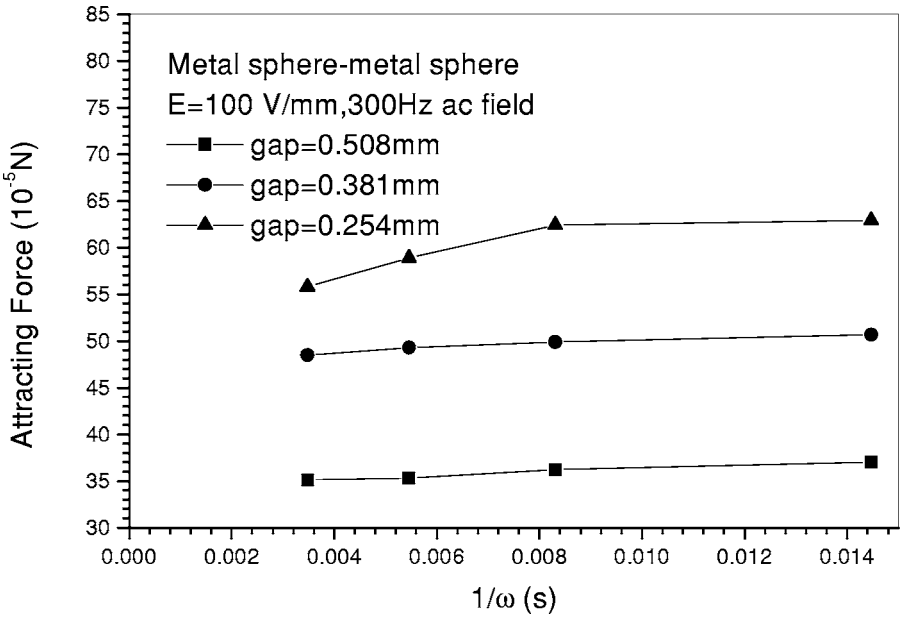

FIG. 4. The attracting force versus $1/\omega$ for two metallic spheres.

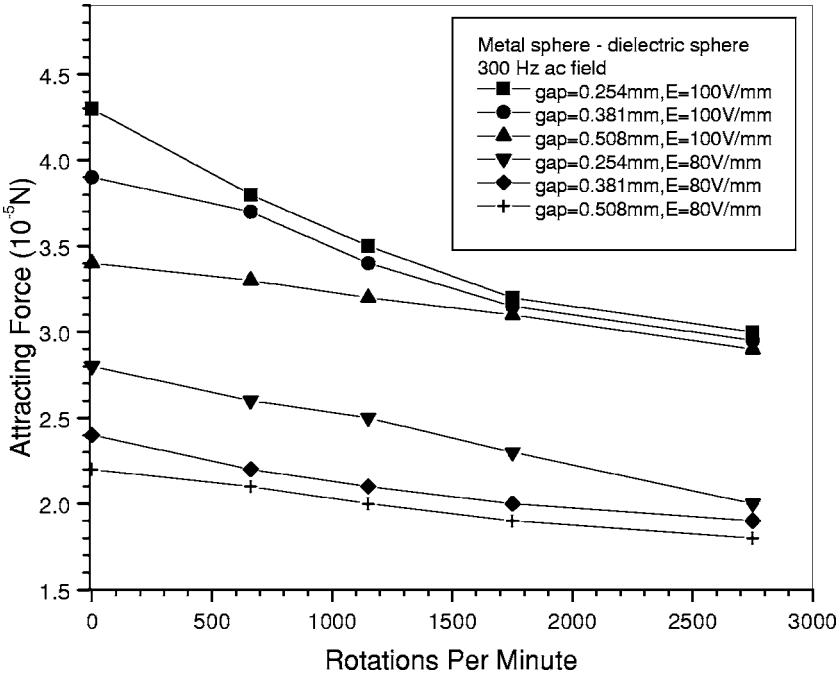

FIG. 5. The attracting force between a rotating metallic sphere and a stationary dielectric sphere versus the rotation speed.

detect any change in the attracting force. On the contrary, the experiment found the reduction was significant. We suggest that when a sphere rotates, the rotation may also weaken the local field at its neighborhood. It is well known in the study of ER fluids that the attracting force between two near-touching conducting spheres is deeply related to the enhanced local field between the gap [15]. The rotation induced reduction of local field seems to be the main reason for the reduction of attracting force with a rotating metallic sphere. This suggestion also explains why the reduction rate is so sensitive to the gap with a rotating metallic sphere. When the rotational sphere is dielectric, the local field is still weakened, but not as significantly as in the case of rotating metallic sphere. Therefore, in the case of a rotating dielectric sphere, the reduction rate of the attracting force is not as sensitive to the gap as in case of a rotating metallic sphere.

The curves in Fig. 8 also suggest that it is incorrect to assume that the polarization just comes from the free surface charge and surface current, which has the relaxation time estimated by Eq. (1.1). In fact, there must be additional polarization due to the molecular polarizability, which has a different relaxation time. Theoretically, if both σ_p and σ_f are vanishing, the spheres can still be polarized due the molecular polarizability. For dielectric materials, a simple harmonic model can be used [16]. The equation of motion for an electron of charge $-e$ and mass m bound by a harmonic force $-m\omega_0^2 r$ and acted on by an electric field E is

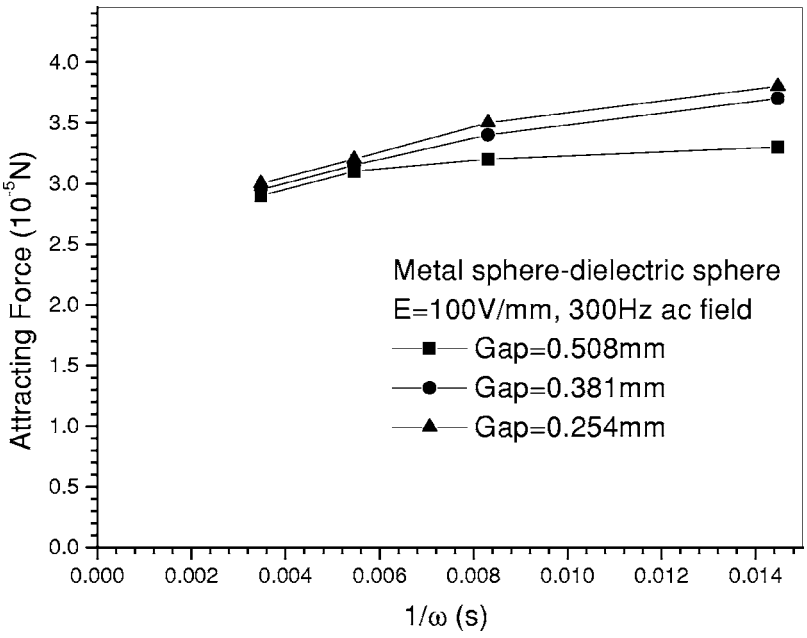

FIG. 6. The attracting force versus $1/\omega$ for a rotating metallic sphere and a stationary dielectric sphere.

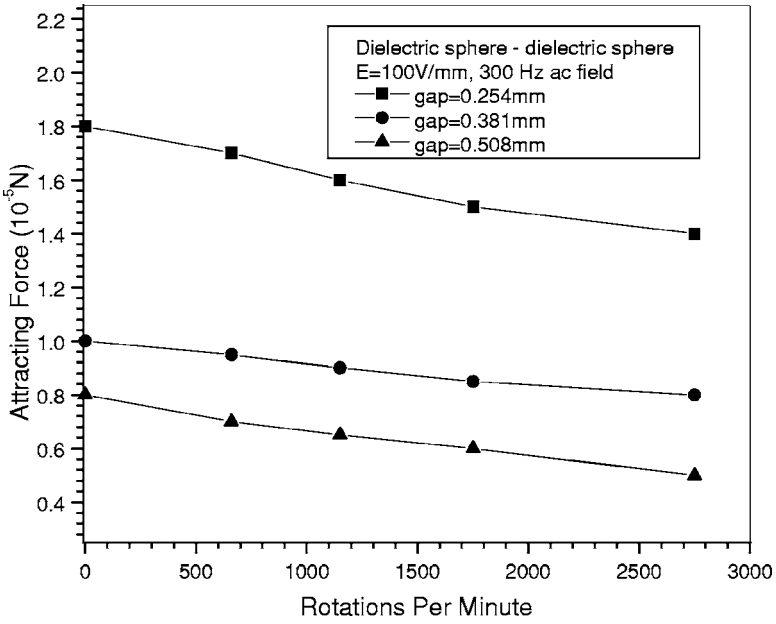

FIG. 7. The attracting force between two dielectric spheres versus the rotation speed.

$$m[r + \gamma r + \omega_0^2 r] = -eE, \tag{4.1}$$

where r is the displacement of the electron from its equilibrium position and γ measures the damping force. This model provides the polarization and the dielectric constant. The relaxation time for this molecular polarization is

$$\tau_0 = 1/\gamma. \tag{4.2}$$

Similar to the derivation in the WYG theory [12], we assume that under a rotation perpendicular to the field direction, the induced dipole from molecular polarization, p_m, is subject to the equation

$$dp_m/dt = \omega \times p_m - (p_m - p_{m0})/\tau_0, \tag{4.3}$$

where p_{m0} is the dipole moment from the molecular polarization before the rotation, which is in the field direction—i.e., the z direction. If we take ω in the x direction, the solution of Eq. (4.3) is given by

$$p_{mz} = p_{m0}/[1 + (\omega\tau_0)^2], \quad p_{mx} = -p_{m0}(\omega\tau_0)/[1 + (\omega\tau_0)^2]. \tag{4.4}$$

On the other hand, the reduced dipole from the free surface charge is given by

$$p_{fz} = p_{f0}/[1 + (\omega\tau)^2], \quad p_{fx} = -p_{f0}(\omega\tau)/[1 + (\omega\tau)^2], \tag{4.5}$$

where τ is given in Eq. (1.1) and p_{f0} is the dipole moment from the free surface charge before the rotation, which is in the z direction.

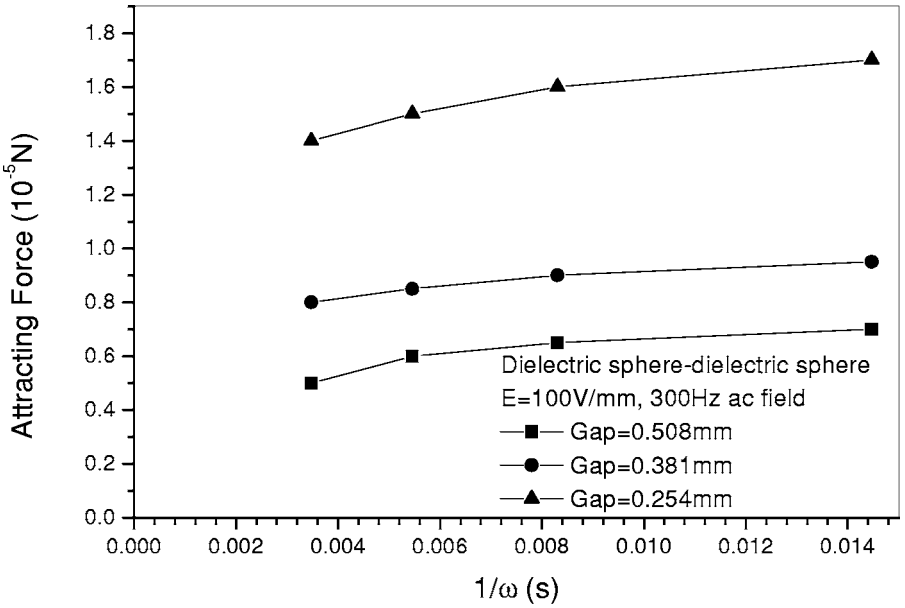

FIG. 8. The attracting force versus $1/\omega$ for two dielectric spheres.

The stationary sphere has the dipole moment $p_t = p_{f0} + p_{m0}$ in the z direction. The attracting force between these two spheres along the z direction is now reduced to

$$f = \frac{f_f}{1 + (\omega\tau)^2} + \frac{f_m}{1 + (\omega\tau_0)^2}, \tag{4.6}$$

where $f_f = 3p_{f0}p_t/(\varepsilon d^4)$ and $f_m = 3p_{m0}p_t/(\varepsilon d^4)$, respectively. Here d is the distance between the two spheres and ε is the dielectric contact of the argon gas. For polyamide materials, τ_0 is estimated around 10^{-4}–10^{-5} s. Then since in our experiment $\omega\tau_0 < 1$, the second term in Eq. (4.6) is almost unchanged with ω while the first term is significantly reduced with ω up to 2750 rpm. This explains the behavior in $\omega \to \infty$ shown in Fig. 8.

We must emphasize that our above explanations and suggestions are preliminary. A correct theory requires combining all above mechanisms together.

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